Dark Energy and Fate of the Universe

Chakkrit Kaeonikhom*

The Institute for Fundamental Study (IF)

Naresuan University, Phitsanulok 65000, Thailand

It has been known that if the equation of state parameter of dark energy is less than minus one, the expansion of the universe shall be rapidly accelerated by a dark energy called "phantom energy". The phantom-energy density increases as the universe expands. It value reaches infinity at a finite time. The phantom energy rips apart everything before the death of the universe in a "big rip."

INTRODUCTION

The mathematical predictions by Friedmann and others within Einstein's general theory of relativity tell us that the universe is not static. Hubble has discovered the expansion of the universe, a strong evidence corresponding to the mathematical predictions. In particular, it has been shown that if the matter that fills the universe can be treated as a pressureless fluid, which would be the case for galaxies, then the universe expands forever (if its spatial geometry is a Euclidean or hyperbolic) or eventually recollapses (if its spatial geometry is of a 3-sphere). But nowadays observations indicating an accelerating expansion of the universe show that this simple model of matter is not sufficient. The universe may consist of some sort of negative-pressure called *dark energy*. Many cosmologists have constructed equation of states of dark energy for explaining and predicting the fate of our universe. One of dark energy models is called *phantom energy* which has equation of state parameter less than minus one.

FRIEDMANN-ROBERTSON-WALKER UNIVERSE

A solution of Einstein's equations describing our universe is called Friedmann-Robertson-Walker metric (FRW metric),

$$\mathrm{d}s^2 = \mathrm{d}t^2 - a^2(t)\left[\frac{\mathrm{d}r^2}{1-kr^2} + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2)\right], \tag{1}$$

where t is the cosmic time, r is the spatial radius coordinate, and k characterizes the curvature of 3-dimensional space of which $k = -1, 0, 1$ corresponds to open, flat and closed universe respectively. Finally, as usual, $a(t)$ is the scale factor. The FRW metric is also based upon assumption of homogeneity and isotropy of the universe. The differential equations for the scale factor and the matter density follow from Einstein's equations:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}, \tag{2}$$

where $G_{\mu\nu}$ is the Einstein tensor, and $R_{\mu\nu}$ is the Ricci tensor which depends on the metric and its derivatives, R is the Ricci scalar and $T_{\mu\nu}$ is the energy-momentum tensor. In the large-scale universe, dynamics of galaxies can be assumed to be like perfect fluid. Therefore the energy-momentum tensor is taken in the form

$$T^{\mu\nu} = (\rho + p) u^\mu u^\nu - p g^{\mu\nu}, \tag{3}$$

where u^μ is 4-velocity, ρ and p are the energy density and the pressure density of the fluid, respectively. Then Eq. (2) gives the two independent equations which are called, respectively, Friedmann's equations and acceleration equation,

$$H^2 \equiv \frac{\dot{a}}{a} = \frac{8\pi G}{3}\rho - \frac{k}{a^2} \tag{4}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p). \tag{5}$$

Hence the Eq. (5) implies that acceleration of the universe occur at $\rho + 3p < 0$. Conservation of energy-momentum tensor leads to the fluid equation

$$\dot{\rho} + 3H(\rho + p) = 0 \tag{6}$$

A useful parameter in cosmology is density parameter, $\Omega(t)$, defined as $\Omega \equiv \rho/\rho_{\rm c}$, where the critical density is defined as $\rho_{\rm c} \equiv 3H^2/8\pi G$. Therefore Eq. (4) can be written in the form

$$\Omega(t) = 1 - \Omega_k(t), \tag{7}$$

where $\Omega_k \equiv -k/a^2H^2$. In summary the matter distribution clearly determines the spatial geometry of our universe, i.e.

$$\begin{aligned}
&\Omega < 1 \quad \text{or} \quad \rho < \rho_{\rm c} \quad \Leftrightarrow \quad k = -1 \quad \Leftrightarrow \quad \text{open universe,}\\
&\Omega = 1 \quad \text{or} \quad \rho = \rho_{\rm c} \quad \Leftrightarrow \quad k = 0 \quad \Leftrightarrow \quad \text{flat universe,}\\
&\Omega > 1 \quad \text{or} \quad \rho > \rho_{\rm c} \quad \Leftrightarrow \quad k = +1 \quad \Leftrightarrow \quad \text{closed universe.}
\end{aligned}$$

EQUATION OF STATE PARAMETER OF DARK ENERGY

In previous section, Eq. (5) implies that acceleration of the universe occuring at $\rho_3 p < 0$ means the pressure must be negative value. If one defines equation of state parameter $w \equiv p/\rho$ then acceleration of the universe obeys $w < -1/3$. For equation of state for matter (baryons plus cold dark matter, sometime called dust), we mention that dynamics of galaxies be like motion of an ideal gas in container. Therefore we can refer to the equation of state of ideal gas, namely $p = \rho_{\rm m} RT$, where the subscript "m" stands for matter. Thus the equation of state parameter for matter simply reads

$$w \equiv \frac{p}{\rho} = \frac{\rho_{\rm m} RT}{\rho_{\rm m} c^2} = \frac{C^2}{c^2}, \tag{8}$$

where $C = \sqrt{RT}$ is *thermal speed* of particles and c is the speed of light (here we set $c \equiv 1$). For matter (slowly moving particles), equation of state can be approximated as $w = 0$.

Another one specie in the universe is radiation or, in the sense of particle, photon. In an ideal model of photon gas, the pressure of photons is given by $p = \rho_\gamma/3$ where ρ_γ is the energy density of photon. Therefore the equation of state for radiation is given by [2] $w = 1/3$. Now we consider evolution of energy density with scale factor, which can be obtained by solving Eq. (6) (assume constant w), that gives

$$\rho \propto a^{-3(1+w)}. \tag{9}$$

Note that Eq. (9) can not be considered for $w = -1$. We can see that, for matter $\rho \propto a^{-3}$, and for radiation $\rho \propto a^{-4}$. At present, radiation may not be supposed to dominate our universe but more possibly maybe the dust in context of species of matter fills in the universe we know. If the universe is filled with matter, then the universe expands forever (if its spatial geometry is Euclidean or hyperbolic) or eventually recollapses (if its spatial geometry is of a 3-sphere). But observations today tells us the spatial curvature of the universe is close to flat geometry [4], and it seems to be accelerated in which the equation of state parameter requires $w < -1/3$. Many have questioned that what sort of energy with $w < -1/3$ causes the accelerating universe? This mysterious energy is assumed to exist and called dark energy.

AN ULTIMATE FATE OF THE UNIVERSE WITH $w < -1$

In this report we focus on equation of state parameter of dark energy of which w is less than minus one. The dark energy called [1] "phantom energy." The Friedmann's equation included energy density of dark energy are taken in the form (neglecting radiation term)

$$H^2(t) = \frac{8\pi G}{3}\left(\rho_{\rm m}(t) + \rho_{\rm DE}(t)\right) - \frac{k}{a^2(t)} \tag{10}$$

or equivalently, in the form of dimensionless density parameter:

$$\Omega_{\rm m} + \Omega_{\rm DE} + \Omega_k = 1, \tag{11}$$

where the subscript "DE" means dark energy. With assuming $\Omega_k = 0$, the Friedmann's equation for phantom energy governing the time t evolution of the scale factor $a(t)$ becomes

$$H^2 = H_0^2 \left[\Omega_{\rm m,0} a^{-3} + (1 + \Omega_{\rm m,0}) a^{-3(1+w)}\right]. \tag{12}$$

Observation of dust ratio today [4] gives $\Omega_{\rm m,0} = 0.273 \pm 0.022$. Then the universe is already dark energy dominated, and for it will become increasingly dark energy-dominated in the future. By solving the Eq. (12), we find that the scale factor blows up at time [1]

$$t_{\rm rip} \simeq t_0 + \frac{2}{3}|1 + w_{\rm DE}|^{-1} H_0^{-1} (1 + \Omega_{\rm m,0})^{-1/2} \tag{13}$$

from the current time t_0, where $t_{\rm rip}$ is the time at the **Big Rip**. If we try to calculate the time at the big rip by using data of $w_{\rm DE}$ from WMAP-7 satellite [5] which gives $w_{\rm DE} = -1.12$, the time at the big rip becomes [3] $t_{\rm rip} = 104.5^{+1.9}_{-2.0}$ billion years. The age of the universe today, roughly approximated 13.8 billion years, means that the end of the universe in this big rip will occur within approximately 91 billion years.

CONCLUSION

Phantom energy is just a model of dark energy with equation of state parameter less than minus one. Observation today [5] yields w in the range crossing minus one, $-1.55 < w < -0.70$, thus phantom model of dark energy can explain some the epoch of the universe in a certain range only. There are many dark energy models for explain the accelerating universe today. However, dark energy needs further explanation.

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* Electronic address: kchakkrit@nu.in.th

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