

# Crossing Examination Paper

The Tah Poe Academia Institute

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## Classical Mechanics I and Classical Mechanics II (100 points)

1. Prove that gravitational force is a conservative force.

**[Hint:** You must show in detail that  $W = \oint \vec{F}_{\text{gravity}} \cdot d\vec{r} = 0$ ].

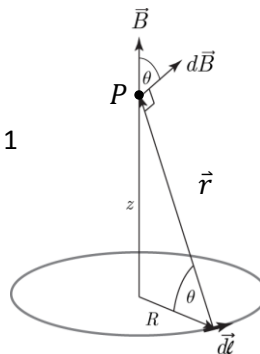
2. Find the center of mass of a half of solid sphere which has uniform mass density  $\rho$ .

**[Hint:** Using the formula  $\vec{R}_{\text{CM}} = \frac{1}{M} \int \vec{r} dm$ ].

## Electrodynamics I and Electrodynamics II (100 points)

3. Find the electric field outside and inside a uniformly charged solid sphere of radius  $R$  and total charge  $Q$ .
4. Find the magnetic field at a distance  $z$  above the center of a circular loop of radius  $R$ , which carries a steady current  $I$  in (Figure 1).

Figure 1



**[Hint:** The Biot-Savart law is given by  $\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{\ell} \times \vec{r}}{r^2}$ ]

## Quantum Mechanics I and Quantum Mechanics II (100 points)

5. A particle of mass  $m$  is in the state

$$\Psi(x, t) = A e^{-a[(mx^2/\hbar)+it]},$$

where  $A$  and  $a$  are positive real constant.

- (a) Find  $A$ .  
 (b) Calculate the expectation values of  $x$  and  $p$ .

Note that this formula,  $\int_{-\infty}^{\infty} e^{-\alpha x^2} dx = \sqrt{\frac{\pi}{\alpha}}$ , may be help you.

6. Suppose that potential of infinite square well is

$$V(x) = \begin{cases} 0, & \text{if } 0 \leq x \leq L, \\ \infty, & \text{otherwise,} \end{cases}$$

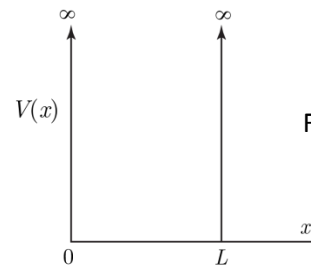


Figure 2

(Figure 2). Solve Schrödinger's equation to obtain the wave function of this system.

## Thermodynamics and Statistical Mechanics (100 points)

7. Prove that, for  $n$  moles of ideal gas, work done in isothermal process is taken as the form

$$W = nRT \ln \left( \frac{V_1}{V_2} \right),$$

where  $R$  is gas constant,  $T$  is temperature and  $V$  is volume of the system.

8. The Maxwell velocity distribution of monatomic gas of mass  $m$  is given by

$$P(v)dv = \frac{4}{\pi^{1/2}} \left( \frac{m}{2k_B T} \right)^{3/2} v^2 e^{(-mv^2/2k_B T)} dv.$$

Show that the mean kinetic energy of monatomic gas is

$$\langle E \rangle = \frac{3}{2} k_B T.$$

[Hint: Find  $\langle v^2 \rangle$  first by using the formula  $\int_0^{\infty} x^4 e^{-ax^2} dx = \frac{3\pi^{1/2}}{8a^{5/2}}$ ].

## Modern Physics (100 points)

9. A total relativistic energy of mass  $m$  moving with speed  $v$  is

$$E = \gamma mc^2 = E_k + mc^2,$$

where  $\gamma = 1 / \sqrt{1 - v^2 / c^2}$ ,  $E_k$  is kinetic energy and  $c$  is speed of light.

(a) Show that the total energy  $E$  can be written in alternative form as

$$E^2 = p^2 c^2 + m^2 c^4,$$

where  $p$  is its relativistic momentum.

(b) Show that for Newtonian limit ( $v \ll c$ ) the kinetic energy reduces to  $E_k \simeq \frac{1}{2}mv^2$ .

10. The energy level of hydrogen atom is given by

$$E_n = -\frac{e^2}{8\pi\epsilon_0 a_0} \frac{1}{n^2} = -\frac{13.6 \text{ eV}}{n^2}, \quad n = 1, 2, 3, \dots$$

where  $e$  is charge of electron,  $a_0 = 4\pi\epsilon_0 \hbar^2 / (e^2 m_e)$  is Bohr's radius,  $m_e$  is mass of the electron,  $\hbar$  is reduced Planck's constant and  $\epsilon_0$  is permittivity of free space.

(a) Derive the energy level of hydrogen atom by beginning from the total energy  $E = E_k + E_p$ .

**[Hint:** 1) Equating Coulomb's force and centrifugal force to obtain kinetic energy  $E_k$ .

2) Using quantization rule of angular momentum  $L_n = mv_n r_n = n\hbar$  ].

(b) Use the Planck relation  $\Delta E = E_{n_i} - E_{n_f} = h\nu$  ( $\nu$  is frequency of a photon) to derive the Rydberg's formula

$$\frac{1}{\lambda} = R_H \left( \frac{1}{n_f^2} - \frac{1}{n_i^2} \right),$$

where the Rydberg's constant is  $R_H = e^2 / (8\pi\epsilon_0 a_0 h c)$ .